# A THEORETICAL STUDY OF $\pi$-HYDROCARBON-IRON TRICARBONYL COMPLEXES 

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#### Abstract

The molecular orbital parameters for tricarbonyl(tetrahapto-unsaturated-hydrocarbon)iron complexes are computed using graph-theoretical methods. The results are in agreement with their experimental properties.


## 1. Introduction

Merging the graph-theoretical methods previously developed by the author [1-8] with the topological Hückel model developed by Mingos [9], one can rapidly compute the characteristic polynomials and eigenvalues of olefin-metal complexes. This leads to a facile noncomputer oriented method for obtaining useful Hückel MO parameters. The ease with which these calculations are performed not only facilitates our chemical thinking processes, but also provides us with a unique perspective in the conceptualization of chemical phenomena. The study of different ways of comparing molecules contributes to our better understanding of their chemistry. Herein, we present a study of carbonylmetal $\eta^{4}$-olefin complexes using these methods. A structureresonance theory study of these metal complexes has been reported [10].

The Mingos model assumes the basis-set orbitals for the $\pi$-electron calculation as being the hydrocarbon polyene $p \pi$ orbitals (i.e. unhybridized carbon $p_{z}$ orbitals) and each of the metal $\mathrm{e}_{x z}\left(\mathrm{~d}_{x z}+\mathrm{p}_{x}=2 \mathrm{e}_{x z}\right)$ and $\mathrm{e}_{y z}\left(\mathrm{~d}_{y z}+\mathrm{p}_{y}=2 \mathrm{e}_{y z}\right)$ hybrid orbitals (fig. 1). The metal and carbon Coulomb integrals ( $\alpha_{\mathrm{c}}=\alpha_{\mathrm{m}}=\alpha$ ) and carbon-carbon $2 \mathrm{p} \pi-2 \mathrm{p} \pi$ and the metal-carbon $\mathrm{e}_{x z}-2 \mathrm{p} \pi$ and $\mathrm{e}_{y z}-2 \mathrm{p} \pi$ exchange integrals ( $\beta_{\mathrm{cc}}=\beta_{\mathrm{cm}}=\beta$ ) are taken to be comparable; the effect of electronegativity differences on $\alpha_{\mathrm{m}}$ can be introduced as an additional refinement later on. The signs of the metal-carbon exchange integrals ( $\beta$ ) are chosen to reflect the phase changes in the chosen set of basis orbitals: $-\beta$ is associated with a phase change (fig. 1 ); a single phase change in a cyclic $\pi$-network is a Möbius system [11]. A double phase change acts like a double negative to give a "normal cycle".

These assumptions describe the metal-polyene fragment as being perturbed to a negligible extent by the carbonyl groups. Justification for this derives from both experimental and theoretical studies [12,13]. The basis orbitals for tricarbonyl ( $\eta^{4}$-butadiene)iron and tricarbonyl ( $\eta^{4}$-cyclobutadiene)iron with their corresponding molecular graphs are depicted in fig. 1 . The iron $d_{x z}-p_{x}$ and $d_{y z}-p_{y}$ hybrid


Fig. 1. Basis orbitals and corresponding orbital network graphs for $\mathrm{Fe}\left(\eta^{4} \text {-unsaturated-hydrocarbon)( } \mathrm{CO}\right)_{3}$ complexes.
orbitals combine with the organic ligand $p_{z}$ orbitals to form a three-dimensional delocalized electronic network containing $N_{\mathrm{c}}$ electrons, where $N_{\mathrm{c}}$ is the total number of basis orbitals. The $\mathrm{Fe}(\mathrm{CO})_{3}$ moiety is characterized by the electronic configuration $\left(\mathrm{e}_{x z}\right)^{1}\left(\mathrm{e}_{y z}\right)^{1}$, with its remaining six metal electrons occupying the non-bonding $\mathrm{d}_{z 2}$, $\mathrm{d}_{x^{2}-y^{2}}$, and $\mathrm{d}_{x y}$ orbitals. Thus, the $\mathrm{Fe}(\mathrm{CO})_{3}$ group contributes two electrons for metal-butadiene bonding.

In this work, an orbital network graph (fig. 1) is a graph where vertices correspond to atomic $\pi$-orbitals and edges (lines) connecting the vertices associate the pairs


номо $=0.6030$
$E_{\pi}=14.4481$
$\Delta E_{\pi}=4.024$


НОМО $=0.4959$
$E=16.8959$
$E_{\pi}=16.8959$
$\Delta E_{\pi}=4.054$



НомО $=0.4977$
$E_{\pi}=17.5475$
$\Delta E_{\pi}=3.864$
$\operatorname{Pe}\left(\eta^{4}-\mathrm{C}_{6} \mathrm{H}_{6}\right)(\mathrm{CO})_{3}$

номо-0.6180
$E_{\pi}=14.4721$
$\Delta E_{H}=4.815$
$\mathrm{Fe}\left(\mathrm{n}^{4}-\mathrm{C}_{1} \mathrm{O}_{8}\right)(\mathrm{CO})_{3}$

НОМО $=0.2624$ $\mathrm{HOMO}=0.2624$
$E_{\pi}=17.3094$
$\Delta E_{\pi}=4.467$
Fig. 2 (continued on following page).

Fig. 2 (continued on following page).

Fig. 2. HMO parameters for various $\mathrm{Fe}\left(\eta^{4}\right.$-unsaturated-hydrocarbon $)(\mathrm{CO})_{3}$ complexes.
of $\pi$-orbital interactions. It should be noted that orbital network graphs and their corresponding computed parameters disregard stereochemistry, and the following two orbital network graphs for $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{4} \mathrm{H}_{6}\right)(\mathrm{CO})_{3}$ give identical characteristic polynomials.


Thus, the computed Hückel MO parameters presented herein disregard the molecular folding found in 10 and 21 (fig. 2) and the existence of cis and trans isomers found in 20 to 24 and 30, and the metal basis orbitals can be rotated to give two different but energetically equivalent alignments with regard to the butadiene basis orbitals. In fig. 2, the site of the tricarbonyliron attachment is indicated by heavy lines.

## 2. Determining characteristic polynomials and eigenvalues with graph theory

Making use of graph-theoretical methods previously published, one can quickly determine the characteristic polynomials and eigenvalues of the tricarbonyliron complexes presented in fig. 2. Table 1 gives a glossary of terms and table 2 gives a summary of the equations that will be used. The tetrahapto-iron complexes of butadiene and cyclobutadiene and compounds 1 to 27 shown in fig. 2 are altemant $\pi$ electronic networks (i.e. contain no odd-sized rings) with bonding/antibonding MO energy levels that are paired and that have characteristic polynomials with only even terms and a tail coefficient which equals the corrected structure count $(\mathrm{CSC}=K)$ to the second power. Herndon's method for determining CSC can be used to obtain the tail coefficient $a_{N}= \pm K^{2}$ of the characteristic polynomial of altemant $\pi$-electronic networks [10]. The second coefficient $a_{2}=-q$ is equal to the negative of the number of $\sigma$-bonds in the orbital network graph of the metal carbonyl complex molecule. The $a_{4}$ and $a_{6}$ coefficients are given by eq. (1) and eq. (2), respectively, in table 2. In the application of these latter two, it should be noted that $r_{4}$ and $r_{6}$ give the algebraic sum of the number of tetragonal and hexagonal rings, respectively, where Möbius ring systems are assigned negative values. Also, $r_{4}$ and $r_{6}$ represent all combinatorial tetragonal and hexagonal circuits, respectively. For example, the octahedron possesses fifteen different tetragonal circuits (each of the twelve octahedron edges is central to one tetragonal circuit and each of the three pairs of opposing vertices is central to a tetragonal circuit), and the cube has sixteen distinct hexagonal circuits (each of the twelve cube edges is central to one hexagonal circuit and each of the four pairs of opposing vertices is central to a hexagonal circuit). It should be noted that eq. (1) and eq. (3) are valid for all graphs, and eq. (2) is valid for all graphs having vertices of degree 3 or less; all the equations in table 2 were derived by the author, although some have less general precursors that were previously presented in the literature [1-8].

Table 1
Glossary of terms

| $a_{4}$ | - fourth coefficient in the characteristic polynomial |
| :--- | :--- |
| $a_{6}$ | - sixth coefficient in the characteristic polynomial |
| $\alpha\left(\alpha_{c}\right)$ | - HMO Coulomb integral (of carbon) |
| $\alpha_{3}$ | - number of branches on a trigonal ring |
| $\alpha_{4}$ | - number of branches on a tetragonal ring |
| $\beta$ | - HMO exchange integral |
| $C_{n}$ | - cycle or circuit of size $n$ |
| $d_{i}$ | - number of vertices of degree $i$ |
| $e(i, j)$ | - number of edges with a vertex of degree $i$ at one end and a vertex of |
| $e_{k}$ | - degree $j$ at the other end |
| $\varepsilon$ | - energy level or HMO eigenvalue |
| $E_{x}$ | - total p $\pi$ energy |
| $G$ | - a molecular or isoconjugate graph |
| $G_{x}$ | - a molecular graph with a single weighted vertex |
| $G_{k}$ | - a molecular graph with a single weighted edge |
| $h$ | - the weight of a heteroatom vertex $\left[h=\left(\alpha_{x}-\alpha_{c}\right) / \beta\right]$ |
| $k$ | - the weight of an edge $\left[k=\beta_{c x} / \beta \beta_{c c}\right]$ |
| $L_{n}$ | - linear polyconjugated system of size $n$ |
| $N\left(N_{c}\right)$ | - number of (carbon atom) vertices |
| $P(G ; X)$ | - characteristic polynomial of a molecular graph |
| $q$ | - number of C-C $\sigma$-bond edges |
| $r_{n}$ | - number of rings (cycles) of $n$ vertices |
| $r_{3}^{\prime}$ | - number of combinatorial pairs of trigonal rings |
| $v$ | - vertex |
| $X$ | $-(\varepsilon-\alpha) / \beta=$ graph eigenvalue |
| $Z_{k}$ | - cycles containing edge $k$ |

To illustrate the use of the equations in table 2 for generating the characteristic polynomials, we will now present the solutions to the graph of the cube and the orbital network graphs of tricarbonyl $\left(\eta^{4}\right.$-cyclobutadiene)iron and tricarbonyl $\left(\eta^{4}\right.$-benzene $)$ iron. For the cube graph $q=12, a_{4}=30, a_{6}=-28$, and $K=3$, which gives $P$ (cube; $X$ ) $=X^{8}-12 X^{6}+30 X^{4}-28 X^{2}+9$ [14]. For the tricarbonyl $\left(\eta^{4}\right.$-cyclobutadiene)iron molecular graph (fig. 1 ), $N_{\mathrm{c}}=6, q=8$, and $e(3,3)=4$. Figure 3 shows all the tetragonal and hexagonal circuits for this molecular graph; there are four Möbius and one regular tetragonal circuits, and two hexagonal circuits crossing over two negative unit weighted edges giving $r_{4}=-4$ with $\alpha_{4}=3, r_{4}=1$ with $\alpha_{4}=4$, and $r_{6}=2$. Inputting these parameters into eqs. (1), (2), and (3) gives the characteristic polynomial for tricarbonyl ( $\eta^{4}$ -

Table 2
Equations for the calculation of characteristic polynomials of molecular graphs
Eq. no.
Equation

$$
\begin{equation*}
a_{4}=(1 / 2)\left(q^{2}-9 q+6 N_{c}\right)-2 r_{4}-d_{1}-d_{4}-3 d_{5}-6 d_{6}-\cdots \tag{1}
\end{equation*}
$$

$$
\begin{align*}
P(G, X) & =X^{N}-q X^{N-2}-2 r_{3} X^{N-3}+a_{4} X^{N-4}-\left[2 r_{5}-2\left(q-3-\alpha_{3}\right) r_{3}\right] X^{N-5}  \tag{3}\\
& +a_{6} X^{N-6}+\ldots
\end{align*}
$$

$$
\begin{equation*}
P\left(G_{x} ; X\right)=X P\left(G_{x}-v_{x} ; X\right)-h P\left(G_{x}-v_{x} ; X\right)-k^{2}\left[X P\left(G_{x}-v_{x} ; X\right)-P(G ; X)\right] \tag{4}
\end{equation*}
$$

for edge/vertex weighted graphs

$$
\begin{equation*}
P\left(G_{k} ; X\right)=P\left(G_{k}-e_{k} ; X\right)-k^{2} P\left(G_{k}-\left(e_{k}\right) ; X\right)-2 k \Sigma P\left(G_{k}-Z_{k} ; X\right) \tag{5}
\end{equation*}
$$

for edge weighted graphs

$$
\begin{gather*}
P\left(G_{0} ; X\right)=P(G ; X)+P\left(G_{0}-v_{0} ; X\right)  \tag{6}\\
\quad \text { for right mirror-plane fragment }
\end{gather*}
$$

$$
\begin{equation*}
P\left(G_{\mathrm{e} 1} ; X\right)=P\left(G_{\mathrm{e} 1}-e_{\omega} ; X\right)-P\left(G_{\mathrm{e} 1}-\left(e_{\omega}\right) ; X\right)+\sum P\left(G_{\mathrm{e} 1}-Z_{\omega} ; X\right) \tag{7}
\end{equation*}
$$

for the identical fragments of vertex-centric 3 -fold graphs
$P\left(G_{\mathrm{e} 2} ; X\right)=P\left(G_{\mathrm{e} 2}-e_{\omega} ; X\right)-P\left(G_{\mathrm{e} 2}-\left(e_{\omega}\right) ; X\right)-\Sigma P\left(G_{\mathrm{e} 2}-Z_{\omega} ; X\right)$ for the identical fragments of ring-centric 3 -fold graphs

$$
\begin{equation*}
P\left(L_{n} ; X\right)=X P\left(L_{n-1} ; X\right)-P\left(L_{n-2} ; X\right), \text { where } P\left(L_{0} ; X\right)=1 \text { and } P\left(L_{1} ; X\right)=X \tag{9}
\end{equation*}
$$

cyclobutadiene)iron as $P(T C C B I ; X)=X^{6}-8 X^{4}+20 X^{2}-16$; the tail coefficients can be verified by using Hemdon's CSC method [10]. Finally, for tricarbonyl ( $\eta^{4}$-benzene)iron ( 1 in fig. 2 ; compare with the orbital network graph in fig. 5), $N_{\mathrm{c}}=8, q=10$, and $e(3,3)=3$. The metal complex 1 has two Möbius tetragonal rings $\left(r_{4}=-2\right)$, each having three branches ( $\alpha_{4}=3$ ) and two Möbius and two regular hexagonal circuits ( $r_{6}=-2$ $+2=0$ ) giving $P(1 ; X)=X^{8}-10 X^{6}+33 X^{4}-37 X^{2}+4$, where the tail coefficient was obtained from the CSC. In this way, all alternant molecular graphs up to $N_{c}=6$ and all alternant molecular graphs with vertices of degree 3 or less up to $N_{c}=8$ can be solved. The solution of larger molecular graphs requires knowledge of some eigenvalues via embedding or by decomposition per eq. (5) into smaller graphs.

The determination of the characteristic polynomial of a non-alternant metal complex is illustrated by 28 , which has $N_{c}=8, q=10, r_{4}=-2, r_{5}=1-1-1=-1$,

(a) All the tetragonal circuits in $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{4} \mathrm{H}_{4}\right)(\mathrm{CO})_{3}$ are shown by the dashed outlines.

(b) All the hexagonal circuits in $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{4} \mathrm{H}_{4}\right)(\mathrm{CO})_{3}$ are shown by the dashed outlines.

Fig. 3. All the tetragonal and hexagonal circuits (dashed outlines) in $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{4} \mathrm{H}_{4}\right)(\mathrm{CO})_{3}$.
$r_{6}=1, r_{7}=2$, and $d_{1}=1$. Application of eqs. (1) to (3) gives $X^{8}-10 X^{6}+32 X^{4}+2 X^{3}$ $-36 X^{2}-6 X+9$, where the tail coefficient can be obtained by the method of Herndon [10] and $a_{7}$ is the sum of $2 C_{7}+2\left(C_{5}+K_{2}\right)$ graph components with sign inversions for Möbius circuits.

Complexes 1, 6, 8, 10, 11, 13, 16, 17, 19, 21, 24, 25, 28, 29 and 30 have no vertices larger than degree 3 , and we will outline how to obtain the characteristic polynomial of some of these complexes. Application of eq. (5) decomposes 6 into fragments that are solved per eqs. (1) to (4) to give

$$
\begin{aligned}
& \text { (4 } \\
& =X^{10}-11 X^{8}+41 X^{6}-57 X^{4}+19 X^{2}-\left(X^{8}-10 X^{6}+33 X^{4}-37 X^{2}+4\right) \\
& =X^{10}-12 X^{8}+51 X^{6}-90 X^{4}+56 X^{2}-4 .
\end{aligned}
$$

The end terms of eq. (5) are zero when one operates on a pendant side chain, as was done for 6 . Successive decomposition of 11 per eq. (5) gives

$$
\begin{aligned}
& \quad-P(6)-2\left(P(L)-2 P\left(L_{2}\right)+X^{2}+1\right) \\
& =\left(X^{2}-1\right)\left(X^{10}-12 X^{8}+50 X^{6}-84 X^{4}+49 X^{2}-9\right)-\left(X^{10}-11 X^{8}+41 X^{6}-57 X^{4}+19 X^{2}\right) \\
& -\left(X^{10}-12 X^{8}+51 X^{6}-90 X^{4}+56 X^{2}-4\right)-2\left(X^{6}-7 X^{4}+14 X^{2}-6\right) \\
& =X^{12}-15 X^{10}+85 X^{8}-228 X^{6}+294 X^{4}-161 X^{2}+25 .
\end{aligned}
$$

Figure 5 summarizes the computation of the characteristic polynomial of 24.

## 3. Kekulé and corrected structure count

While it is well known that the square root of the tail coefficient to the characteristic polynomial of an alternant (bipartite) molecular graph equals the difference in the number of structures of positive and negative parities (CSC), it has also been explicitly recognized that if the tail coefficient of a characteristic polynomial corresponding to a graph has an integral square root, then this value can give the difference in the number of structures of positive and negative parities, regardless of whether the graph is bipartite or not, under the following conditions. Deletion of a vertex from a nonaltemant graph to give an odd altemant graph possessing a nonbonding molecular orbital (NBMO) can have the unnormalized integral coefficients of this orbital written by inspection $[10,15]$. The difference in the number of structures of positive and negative parities (CSC) to the non-alternant graph is given by the algebraic sum of these coefficients adjacent to the deleted vertex. The square of this number $\left[a_{N}= \pm(\operatorname{CSC})^{2}\right]$ gives the tail coefficient to the characteristic polynomial of the corresponding nonalternant graph (cf. complexes 28-30). Also, the sum of the squares of the unnormalized integral coefficients to the NBMO of an odd alternant molecular graph equals the tail coefficient to the corresponding characteristic polynomial; e.g. $G_{2}$ in fig. 4 gives 6 for the tail coefficient. For totally aromatic molecular graphs, the number of Kekule structures equals the corrected structure count ( $K=\mathrm{CSC}$ ). Herein, we make no notational distinction between $K$ and CSC.

Another method for determining $K$ values of a molecular graph $G$ uses the following relationship

$$
K(G)= \pm K(G-e) \pm K(G-(e))
$$

where $G-e$ is the graph obtained upon deletion of some edge $e, G-(e)$ is the corresponding graph obtained upon deletion of edge $e$ with its associated vertices, and the negative sign is only chosen to give a positive (or zero) $K(G)$ value when $e$ belongs to an anti-aromatic ring [ 16,17 ]. By knowing a few $K$ values of some small molecular


Fig. 4. Embedding fragments of $\mathrm{Fe}\left(\eta^{4} \text {-unsaturated-hydrocarbon)( } \mathrm{CO}\right)_{3}$ complexes.
(a) Embedding fragments. (b) Ethene embedding on $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{4} \mathrm{H}_{6}\right)(\mathrm{CO})_{3}$.
graphs, one can use this relationship to obtain $K$ values for larger molecular graphs. In this work, it is worthwhile to remember these values for the tricarbonyliron complexes of butadiene $(K=3)$, cyclobutadiene ( $K=4$ ), and benzene ( $K=2$ ). To illustrate, consider complex 11 as follows:


We denote the molecular graphs of ethene $\left(L_{2}\right)$, allyl $\left(L_{3}\right)$, butadiene $\left(L_{4}\right)$, cyclobutadiene ( $C_{4}$ ), and benzene $\left(C_{6}\right)$ by their standard graph-theoretical designations. Even carbon polyene side chains on a ring system do not change the $K$ value of the ring, as can be noted by comparing 1 versus 6 and tricarbonyl ( $\eta^{4}$-cyclobutadiene)iron versus 5 .




$=X^{2}\left(X^{4}-10 X^{6}+33 X^{4}-37 X^{2}+4\right)-4 X\left(X^{7}-8 X^{3}+20 X^{3}-15 X\right)+4\left(X^{6}-7 X^{4}+15 X^{2}-9\right)$
$=X^{10}-14 X^{4}+69 X^{6}-145 X^{4}+124 X^{2}-36$
$\pm 0.81371$
$\pm 0.93680$
$\pm 1.65789$
$\pm 1.95868$
$\pm 2.42389$

Fig. 5. Determination of eigenvalues by mirror plane fragmentation.

The $K$ value of the acene complexes is given by $K$ (acene) $=3 r_{6}-1$, as can be seen by comparing 1, 11, and 19. Another representative application of this equation gives $K(\mathbf{2 5})=K(19)+K(11)=K(8) \cdot K\left(\mathrm{C}_{10} \mathrm{H}_{8}\right)+K(1) \cdot K\left(C_{6}\right)=13$, where the first equality was obtained by operating on edge $b$ and the second by operating simultaneously on edges $h$ and $p$ of 25. As a final example, consider 14: $K(14)=-K(2)+K($ TCCBI ) $=-1+4=3$, where the minus sign is the result of operating on the edge ( $e$ ) of the antiaromatic cyclooctatetraene ring. Note that in these examples, we avoided operating on the negative weighted edges associated with the attached tricarbonyliron moiety.

## 4. Eigenvalues by embedding

A molecule possessing a subset of eigenvalues found in another molecule is said to be subspectrally related to it $[2,18]$. If a smaller molecule can be embedded onto a larger molecule, then the eigenvalues of the smaller one will be found among the eigenvalues of the larger one [2,19]. To embed a fragment onto an alternant molecule, (a) all atoms connected directly to the fragment must be nodes (have zero coefficients in the corresponding eigenvectors), (b) on the other side of each of these nodes there will
be a repetition of the fragment with the opposite sign, and (c) other branches at these nodes will also be nodes. Common embedding fragments include ethene $(\varepsilon= \pm 1.0 \beta)$, allyl $(\varepsilon= \pm \sqrt{2} \beta)$, 1, 3-butadiene $[\varepsilon= \pm 0.5(\sqrt{5} \pm 1)]$, and pentadienyl $(\varepsilon= \pm 1.0, \pm \sqrt{3})$. Mixed embedding requires that the different fragments have some common eigenvalues, like ethene and pentadienyl $(\varepsilon= \pm 1.0)$. In this work, mixed embedding with the fragments shown in fig. 4 was observed. Tricarbonyl ( $\eta^{4}$-butadiene)iron and the metal complexes $\mathbf{3}, 9,10,12$, and 17 (fig. 2) all are embeddable by ethene and have, therefore, eigenvalues of $\varepsilon= \pm 1.0 \beta$. Mixed embedding of butadiene and $G_{1}$ in fig. 4 occurs in the metal complexes of $8,10,19$, and 20 , which have eigenvalues of $\varepsilon= \pm 0.5(\sqrt{5} \pm 1) \beta$. For allyl, $G_{2}$, or mixed allyl and $G_{2}$ embedding to be possible, the tail coefficient or $K=\operatorname{CSC}$ of a test structure must be divisible by 2 . Mixed allyl and $G_{2}$ embedding of $7(K=4)$ and $G_{2}$ embedding of $21(K=2)$ occurs; since there are two distinct $G_{2}$ embeddings on 21, it is doubly degenerate in the eigenvalues of $\varepsilon= \pm \sqrt{2} \beta$. For pentadienyl, $G_{2}$, or mixed pentadienyl and $G_{2}$ embedding to occur, the test structure must have a $K$ value divisible by 3 . Two distinct mixed embeddings of pentadienyl and $G_{2}$ occur on $14(K=3)$, which is doubly degenerate in eigenvalues of $\varepsilon= \pm \sqrt{3} \beta$. Molecular graphs that have a doubly degenerate eigenvalue subset can have any vertex atom deleted, replaced, or augmented by a polyene substituent, giving successor molecular graphs still retaining this eigenvalue subset once. The tricarbonyliron complexes of $s$-cis-1, 3-butadiene and cyclobutadiene are doubly degenerate in eigenvalues of $\varepsilon= \pm \sqrt{3}, \pm \sqrt{3} \beta$ and $\varepsilon= \pm \sqrt{2}, \pm \sqrt{2} \beta$, respectively, and deletion of a carbon vertex from either can generate molecular graph $G$ in fig. 4 , which has $\varepsilon= \pm \sqrt{2}, \pm \sqrt{3} \beta$. Attachment of a vinyl substituent to tricarbonyl $\left(\eta^{4}\right.$-cyclobutadiene)iron gives 5 , which has $\varepsilon= \pm \sqrt{2} \beta$. Complexes $8,10,20$, and 26 , which are mixed embeddable by $G_{1}$ and $L_{4}$, all have the same HOMO $=0.6180 \beta$ value, and 24 can be embedded by 1 and both have the same $\mathrm{HOMO}=0.3473 \beta$ value .

## 5. $\Delta E_{\pi}$ as a relative measure of reaction spontaneity

It is assumed in this work that the reaction of a polyene to form a tricarbonyliron complex will have its relative spontaneity govemed principally by the change in $\pi$-electronic delocalization energy. Thus, in the following reaction scheme $\Delta E_{\pi}$ $=E_{\pi}\left[\right.$ polyene $\left.-\mathrm{Fe}(\mathrm{CO})_{3}\right]-E_{\pi}[$ polyene $]$ is a relative measure of the tendency for the reaction to take place:

$$
\text { polyene }+\mathrm{Fe}(\mathrm{CO})_{3} \rightarrow \text { polyene }-\mathrm{Fe}(\mathrm{CO})_{3} .
$$

There are two basic kinds of (tetrahapto-polyene)metal complexes. These are typified by the known $s$-cis-1,3-butadiene and cyclobutadiene complexes of tricarbonyliron (fig. 1). The large difference in the $\Delta E_{\pi}$ values between tricarbonyl $\left(\eta^{4}\right.$ butadiene)iron ( $\Delta E_{\pi}=4.456 \beta$ ) and tricarbonyl $\left(\eta^{4}\right.$-cyclobutadiene) iron ( $\Delta E_{\pi}=5.657 \beta$ ) is consistent with the three-dimensional aromatic properties [20] exhibited by the latter metal complex [9]. The carbon-carbon bond lengths in tricarbonyl ( $\eta^{4}$-cyclobutadiene)iron
are all equal, and this complex is known to undergo electrophilic substitution and to resist Diels-Alder reactions. Of all the $\mathrm{Fe}\left(\eta^{4}-\mathrm{C}_{10} \mathrm{H}_{8}\right)(\mathrm{CO})_{3}$ isomers (fig. 2), only 14 is known, which is consistent with its larger $\Delta E_{\pi}$ value [21].

Consider the isomer group 1 to 5 . Only 3 and 5 have been synthesized [21]. The hypothetical reaction of vinylcyclobutadiene with $\mathrm{Fe}(\mathrm{CO})_{3}$ should lead to 5 over 4 because of the larger $\Delta E_{\pi}$ for the former. Similarly, 1, 2-dimethylenylcyclobutadiene would preferentially form 3 over 2 because of its larger $\Delta E_{\pi}$. Since the synthesis of 1 and 11 has not been accomplished, whereas 19 has been synthesized, it appears that for tetrahapto complexes of tricarbonyliron to form, $\Delta E_{\pi}$ needs to be greater than 3.9. In the isomer group of 6 to 10 , only 7,9 , and 10 have been prepared. Again, according to the relative $\Delta E_{\pi}$ values, $\mathrm{Fe}(\mathrm{CO})_{3}$ should react with ortho-quinodimethane to preferentially form 9 over 8. Similarly, 7 should preferentially form over 6. In the isomer group 11 to 14 , only 13 and 14 are known to exist, which is consistent with their higher $\Delta E_{\pi}$ values. Isomers 15 to 18 are all known to exist, although 17 is only a fluxional transient species of 18. The bis-tricarbonyliron complexes of 20 to 23 are known, but 24 is not known, which is consistent with its $\Delta E_{\pi}$ being below 3.9. The synthesis of tricarbonyliron complexes of $\mathbf{2 5}$ to $\mathbf{3 0}$ has been reported. Complex 25 is an example of a carcinogen, and complexes 28 to 30 are examples of non-alternant structures. Complex 27 has the second largest $\Delta E_{\pi}$ value in fig. 2 and can be regarded as the analog of triphenylene, which is the most stable $\mathrm{C}_{18} \mathrm{H}_{12}$ benzenoid isomer, the uncomplexed hydrocarbon corresponding to 27 is not known and is predicted to be reactive since its HOMO value is $0.0835 \beta$ and its $E_{\pi}=21.6928 \beta$, which are smaller than those of its synthesized isomers. Similarly, 26 is the electronic analog of naphthalene, and its complexation with another tricarbonyliron leads to ferraindene coproducts that result from cyclobutadiene ring rupture.

The HOMO values for the tricarbonyliron complexes given in fig. 2 should be useful in predicting the relative ease of electrophilic and Diel s-Alder reactions of their associated organic moieties. It was thought that the Diels-Alder reaction $\left(400{ }^{\circ} \mathrm{C}\right)$ of 9 with methyl propynoate is preceded by dissociation to o-quinodimethane. However, isomerization 9 to 8 followed by Diels-Alder condensation and subsequent decomposition of the resulting tricarbonyliron complex is also plausible. In fact, this latter is even more reasonable in the light of the observation that 9 undergoes pyrolysis to benzocyclobutene of $500^{\circ} \mathrm{C}$. The more facile Friedel-Crafts acetylation of 9 over benzene is consistent with its HOMO value of $0.7046 \beta$ versus $1.0 \beta$ for benzene. The $\mathrm{Fe}(\mathrm{CO})_{3}$ group in acetylergosterol complexes successfully protects the conjugated diene unit, allowing reactions of the free double bond on the 17 -side chain. This result suggests that the HOMO value for the tricarbonyl ( $\eta^{4}$-butadiene)iron unit should be equal to or greater than the HOMO value of $1.0 \beta$ for ethene.

## 6. Summary

The diene-tricarbonyliron complexes constitute a very large class of organoiron compounds. While reviewing polycyclic member compounds of this chemical class, we

Table 3
Characteristic polynomials of $\pi$-hydrocarbon-iron-tricarbonyl complexes

| Cpd | $P(G ; X)-X^{N}+q X^{N-2}$ |
| ---: | :--- |
| 1 | $+33 X^{4}-37 X^{2}+4$ |
| 2 | $+31 X^{4}-31 X^{2}+1$ |
| 3 | $+29 X^{4}-24 X^{2}+4$ |
| 4 | $+30 X^{4}-28 X^{2}+4$ |
| 5 | $+33 X^{4}-42 X^{2}+16$ |
| 6 | $+51 X^{6}-90 X^{4}+56 X^{2}-4$ |
| 7 | $+51 X^{6}-94 X^{4}+72 X^{2}-16$ |
| 8 | $+50 X^{6}-84 X^{4}+49 X^{2}-9$ |
| 9 | $+50 X^{6}-88 X^{4}+65 X^{2}-16$ |
| 10 | $+52 X^{6}-97 X^{4}+72 X^{2}-16$ |
| 11 | $+85 X^{8}-228 X^{6}+294 X^{4}-161 X^{2}+25$ |
| 12 | $+81 X^{8}-190 X^{6}+186 X^{4}-72 X^{2}+9$ |
| 13 | $+83 X^{8}-208 X^{6}+231 X^{4}-94 X^{2}+9$ |
| 14 | $+85 X^{8}-229 X^{6}+295 X^{4}-150 X^{2}+9$ |
| 15 | $+113 X^{10}-379 X^{8}+685 X^{6}-656 X^{4}+301 X^{2}-49$ |
| 16 | $+114 X^{10}-386 X^{8}+701 X^{6}-668 X^{4}+301 X^{2}-49$ |
| 17 | $+114 X^{10}-385 X^{8}+692 X^{6}-639 X^{4}+259 X^{2}-25$ |
| 18 | $+113 X^{10}-378 X^{8}+679 X^{6}-644 X^{4}+292 X^{2}-49$ |
| 19 | $+162 X^{12}-687 X^{10}+1643 X^{8}-2219 X^{6}+1597 X^{4}-532 X^{2}+64$ |
| 20 | $+99 X^{8}-298 X^{6}+447 X^{4}-300 X^{2}+64$ |
| 21 | $+100 X^{8}-308 X^{6}+480 X^{4}-336 X^{2}+64$ |
| 22 | $+129 X^{10}-474 X^{8}+952 X^{6}-1024 X^{4}+531 X^{2}-100$ |
| 23 | $+98 X^{8}-289 X^{6}+423 X^{4}-281 X^{2}+64$ |
| 24 | $+242 X^{14}-1334 X^{12}+4373 X^{10}-8670 X^{8}+10093 X^{6}-6356 X^{4}+1828 X^{2}-144$ |
| 25 | $+264 X^{16}-1542 X^{14}+5476 X^{12}-12233 X^{10}+17184 X^{8}-14736 X^{6}+7232 X^{4}-1797 X^{2}+169$ |
| 26 | $+60 X^{6}-122 X^{4}-104 X^{2}-25$ |
| 27 | $+219 X^{14}-1132 X^{12}+3491 X^{10}-6635 X^{8}+7734 X^{6}-5307 X^{4}+1942 X^{2}-289$ |
| 28 | $+32 X^{4}+2 X^{3}-36 X^{2}-6 X+9$ |
| 29 | $+114 X^{10}-384 X^{8}+680 X^{6}+2 X^{5}-596 X^{4}-10 X^{3}+209 X^{2}+12 X-16$ |
| 30 | $+182 X^{12}-842 X^{10}+4 X^{9}+223 X^{8}-44 X^{7}-3414 X^{6}+176 X^{5}+2777 X^{4}-300 X^{3}-981 X^{2}$ |
|  | $+180 X+64$ |

have shown how to rapidly determine their Hückel molecular parameters using chemical graph theory without the aid of a computer or group theory. This work contributes further toward the comprehensive application of graph theory in the determination of the characteristic polynomials and eigenvalues of molecules.

It has been shown that a difference in the electronic $\mathrm{p} \pi$ energy of the complex and reactant polyene ( $\Delta E_{\pi}$ ) reliably predicts the chemistry of diene-tricarbonyliron complexes. These results are in complete agreement with localization energy calcu-
lations [22] associated with the polyene reactant and with structure-resonance theory calculations associated with the complex [10]. The critical value of $\Delta E_{\pi}$ below which synthesis of a diene-tricarbonyliron complex appears to be difficult is $-3.9 \beta$.

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